The Congruence Energy: A Contribution to Nuclear Masses, Deformation Energies and Fission Barriers *

W.D. Myers and W.J. Swiatecki

It has been know for a long time that nuclear binding energies, when plotted along isobaric sequences that cross the N = Z locus, exhibit (after correction for even-odd effects) a slope discontinuity roughly proportional to $|I|$, where $I = (N-Z)/A$.

Our interpretation of a negative contribution with a kink at $N = Z$ is related to the quantal granularity of nucleonic densities. Thus the density distribution of a quantized particle in a potential well consists of cushion-like bumps boxed in by a latticework of the wave function nodal surfaces. A pair of nucleons with congruent nodal surfaces, say a neutron and a proton, will interact more strongly (in the case of short range forces) than a pair with uncorrelated density modulations. Since the number of neutron-proton pairs is the lesser of N and Z, i.e., $\frac{1}{2}\,(N+Z) - \frac{1}{2}\,|N-Z|$, and since each pair interacts with a strength proportional to the reciprocal of the nuclear volume, i.e., to A^{-1}, the congruence energy should contain a negative term independent of A, modified by a positive term proportional to $|N-Z|/A$. A correction of this type should thus be added to a Thomas-Fermi model, whose smooth, structureless density distributions ignore this rather obvious consequence of quantization in a finite potential well.

A remarkable feature of the congruence energy, in addition to its telltale dependence on $|I|$, is that, being independent of A, it has the same value for a fissioning nucleus as for each of the resulting fission fragments. Hence the total congruence energy must somehow double as the fissioning nucleus deforms into a necked-in scission shape. There is empirical evidence for such doubling.

Figure 2 in Ref. [3] shows the fission barriers of 36 nuclei as a function of a fissility parameter. Two curves in this figure connect points calculated using the above-mentioned Thomas-Fermi model. The upper one assumed that the congruence energy is the same at the saddle point as in the ground state, the lower that it has doubled. There is a fascinating hint in the fact that the almost perfect agreement with the upper curve for Radium and heavier elements, gives place to an approach to the lower curve for lighter nuclei. The Radium region is precisely where saddle-point shapes develop (rather suddenly) a pronounced neck!

We have developed a model for the shape dependence of the congruence energy according to which the congruence energy is linear in the neck radius, and doubles its original value as the neck tends to zero. We have incorporated this shape dependence into our Thomas-Fermi calculations of nuclear deformabilities and fission barriers. The method we use is to solve the Thomas-Fermi equations first for a spherical nucleus and then for a sequence of elongating configurations for which the separation between the centers of mass of the two reflection symmetric halves is constrained to increase by a distance 2D, where D = 0.2, 0.4, 0.6 . . . fm. The resulting fission barriers are in good agreement with measurements.

*Extracted from Ref. [3]

[1] J.M. Blatt and V.F. Weisskopf, Theoretical Nuclear Physics, John Wiley & Sons, New York, 1952.

[2] W.D. Myers and W.J. Swiatecki, Nucl. Phys. 81 (1966) 1.

[3] W.D. Myers and W.J. Swiatecki, LBL-39224, November, 1995.